

LECTURE 42

Recall that $D_v f(x_0, y_0) = D_u f(x_0, y_0) = f_x(x_0, y_0)u_1 + f_y(x_0, y_0)u_2$

Notice that this is $\langle f_x(x_0, y_0), f_y(x_0, y_0) \rangle \cdot u$



If f has first partials at (x_0, y_0) , then the gradient of f , denoted $\nabla f(x_0, y_0)$ is the vector

called grad f $\nabla f(x_0, y_0) = \langle f_x(x_0, y_0), f_y(x_0, y_0) \rangle$

∇ is an operator acting on functions

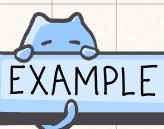


at each point (x, y) , then $\nabla f(x, y)$ gives the direction of the maximum slope of the surface $z = f(x, y)$, and the maximum slope is $\|\nabla f(x, y)\|$. $-\nabla f(x, y)$ gives the minimum slope



If f is differentiable at (x_0, y_0) and $\nabla f(x_0, y_0) \neq 0$ then $\nabla f(x_0, y_0)$ is normal to the level curve of $f @ (x_0, y_0)$

Remember that ∇f points in the direction that f increases most rapidly



$z = 2000 - 2x^2 - 4y^2$ and a hiker is at Mount Calc at $(20, 5, 1100)$

1) Steepest route, what direction? 2) Same height, what direction?

1) move in $\nabla f(x, y) = \langle f_x(20, 5), f_y(20, 5) \rangle$.

$$\frac{\partial}{\partial x} = -4x \quad \frac{\partial}{\partial y} = -8y, \text{ so } \nabla f(x, y) = \langle -80, -40 \rangle = 40 \langle -2, -1 \rangle$$

2) orthogonal to ∇z give level curves, so $\langle -1, 2 \rangle$ and $\langle 1, -2 \rangle$

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Lecture 42 Problems

1) $f(x,y) = x \cos y + y \sin x$. which vect. perp to level curve $f(x,y)=0$ @ $(0,0)$ orthogonal to $\nabla f(x,y)$. $\nabla f = \langle f_x(0,0), f_y(0,0) \rangle$

$$\frac{\partial}{\partial x} x \cos y + y \sin x = \cos y + y \cos x \quad \nabla f = \langle 1, 0 \rangle \quad \boxed{A}$$

$$\frac{\partial}{\partial y} x \cos y + y \sin x = -x \sin y + \sin x$$

2) same as example, C

3) $f(0,0)=0$ and for all (x,y) w/ $x^2+y^2 \leq 1$, $\|\nabla f(x,y)\| \leq 3$

$$\nabla f(x,y) = \langle f_x, f_y \rangle$$