11.27.22

LECTURE 42

Recall that $D_v f(x_0,y_0) = D_u f(x_0,y_0) = f_x(x_0,y_0)u_1 + f_y(x_0,y_0)u_2$

Notice that this is $\langle f_x(x_0,y_0), f_y(x_0,y_0)\rangle \cdot u$

DEFINITION

If f has first partials at (x_0,y_0), then the gradient of f, denoted $\nabla f(x_0,y_0)$ is the vector

called grad f

$$\nabla f(x_0,y_0) = \langle f_x(x_0,y_0),\ f_y(x_0,y_0)\rangle$$

∇ is an operator acting on functions

THEOREM

at each point (x,y), then $\nabla f(x,y)$ gives the direction of the maximum slope of the surface $z=f(x,y)$, and the maximum slope is $\|\nabla f(x,y)\|$. $-\nabla f(x,y)$ gives the minimum slope

THEOREM

If f is diffrentiable at (x_0,y_0) and $\nabla f(x_0,y_0) \neq 0$ then $\nabla f(x_0,y_0)$ is normal to the level curve of f @ (x_0,y_0)

Remember that ∇f points in the direction that f increases most rapidly

EXAMPLE

$z = 2000 - 2x^2 - 4y^2$ and a hiker is at Mount Calc at $(20,5,1100)$
1) Steepest route, what direction? 2) Same height, what direction?

1) move in $\nabla f(x,y) = \langle f_x(20,5), f_y(20,5)\rangle$.

$\frac{\partial}{\partial x} = -4x \quad \frac{\partial}{\partial y} = -8y$, so $\nabla f(x,y) = \langle -80,-40\rangle = 40\langle -2,-1\rangle$

2) orthogonal to ∇z give level curves, so $\langle -1,2\rangle$ and $\langle 1,-2\rangle$

LECTURE 42

Lecture 42 Problems

1) $f(x,y) = x\cos y + y\sin x$. which vect. perp to level curve $f(x,y)=0$ @ 0,0
orthogonal to $\nabla f(x,y)$. $\nabla f = \langle f_x(0,0), f_y(0,0)\rangle$

$\frac{\partial}{\partial x} x\cos y + y\sin x = \cos y + y\cos x$ $\quad \nabla f = \langle 1, 0\rangle$ $\quad \boxed{A}$

$\frac{\partial}{\partial y} x\cos y + y\sin x = -x\sin y + \sin x$

2) same as example, C

3) $f(0,0)=0$ and for all (x,y) w/ $x^2+y^2 \leq 1$, $\|\nabla f(x,y)\| \leq 3$

$\nabla f(x,y) = \langle f_x, f_y\rangle$